OPTIMIZATION OF MLS RECEIVERS
FOR MULTIPATH ENVIRONMENTS

Semi-Annual Report

NASA Grant NSG 1128

Submitted to:

NASA Scientific & Technical Information Facility
P. O. Box 8757
Baltimore/Washington International Airport
Maryland 21240

Submitted by:

G. A. McAlpine
Associate Professor

J. H. Highfill
Senior Engineer

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

RESEARCH LABORATORIES FOR THE ENGINEERING SCIENCES

UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901

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I. INTRODUCTION

This is an interrim report of the second phase of research under Grant NSG 1128, dealing with the design of a Microwave Landing System (MLS) aircraft receiver capable of optimal performance in the multipath environments found in air terminal areas. The project focuses on the angle-tracking problem of the MLS receiver; the work reported here includes tracking system design considerations, continued study and application of locally optimum estimation, involving multipath adaptive reception and then envelope processing, and finally microcomputer system design considerations. A significant result obtained is that envelope processing is competitive in this application with i-f signal processing performance-wise and is much simpler and cheaper, hence future effort will focus on envelope processing. To provide a basis for discussing the results obtained, a brief summary of the signal model concludes this introductory section.

In the last report [1] the received signal in the (linear) i-f channel was modeled as a function $y(t_k,\tau)$ of a global discrete-time variable t_k and a continuous time variable τ local to the present scan. as follows:

$$y(t_k,\tau) = y_D(t_k,\tau) + y_R(t_k,\tau) + n(\tau) \qquad \text{(I-1)}$$

corresponding to direct-path and reflected components and receiver noise, respectively where, neglecting doppler effects, on the kth received scan and for $0 \le \tau \le T$,

$$y_D(t_k,\tau) = \alpha(t_k)p[\theta_A(\tau) - \theta(t_k)]\cos[\omega_{IF}\tau + \beta(t_k)] \qquad \text{(I-2)}$$

$$y_R(t_k,\tau) = \sum \alpha_i(t_k,\tau)p[\theta_A(\tau - \frac{\Delta r_i(t_k)}{c}) - \theta_{R_i}(t_k)]\cos[\omega_{IF}\tau + \beta_i(t_k)] \qquad \text{(I-3)}$$

$$= y_{R_c}(t_k,\tau)\cos[\omega_{IF}\tau + \beta(t_k)] - y_{R_s}(t_k,\tau)\sin[\omega_{IF}\tau + \beta(t_k)] \qquad \text{(I-4)}$$

$n(\tau)$ = stationary, bandpass, Gaussian process, mean zero, variance σ_n^2 (I-5)

$$= n_c(\tau)\cos[\omega_{IF}\tau + \beta(t_k)] - n_s(\tau)\sin[\omega_{IF}\tau + \beta(t_k)] \quad \text{(I-6)}$$

Alternatively, in terms of the above, we may write also

$$y(t_k,\tau) = V(t_k,\tau)\cos[\omega_{IF}\tau + \beta(t_k) - \Gamma(t_k,\tau)] \quad \text{(I-7)}$$

where

$$V(t_k,\tau) = \sqrt{\{\alpha(t_k)p[\theta_A(\tau)-\theta(t_k)]+y_{R_c}(t_k,\tau)+n_c(\tau)\}^2+\{y_{R_s}(t_k,\tau)+n_s(\tau)\}^2} \quad \text{(I-8)}$$

$$\Gamma(t_k,\tau) = \text{arc tan}\{\frac{y_{R_s}(t_k,\tau)+n_s(\tau)}{\alpha(t_k)p[\theta_A(\tau)-\theta(t_k)]+y_{R_c}(t_k,\tau)+n_c(\tau)}\} \quad \text{(I-9)}$$

$$y_{R_c}(t_k,\tau) = \sum_i \alpha_i(t_k,\tau)p[\theta_A(\tau - \frac{\Delta r_i(t_k)}{c}) - \theta_{R_i}(t_k)]\cos[\beta_i(t_k) - \beta(t_k)] \quad \text{(I-10)}$$

$$y_{R_s}(t_k,\tau) = \sum_i \alpha_i(t_k,\tau)p[\theta_A(\tau - \frac{\Delta r_i(t_k)}{c}) - \theta_{R_i}(t_k)]\sin[\beta_i(t_k) - \beta(t_k)] \quad \text{(I-11)}$$

and

$$\beta_i(t_k) - \beta(t_k) = \beta_i(t_{k-1}) - \beta(t_{k-1}) - \frac{\omega_c}{c}(t_k - t_{k-1})\Delta\dot{r}_i(t_k). \quad \text{(I-12)}$$

Refe ence to [1] is made for definitions of unfamiliar quantities not defined (or redefined) above. The principal parameter to be estimated is the aircraft (A/C) angular coordinate $\theta(t_k)$; other parameters whose estimates are needed for the θ-estimation include the amplitude parameter $\alpha(t_k)$ and the noise variance σ_n^2. Necessary modeling of the evolutionary dynamics of θ and α has been deferred until the requirements of the estimation algorithm chosen become firm.

II. TRACKING ALGORITHM DEVELOPMENT

The sought tracking algorithm for the MLS receiver is a discrete-time estimator of a vector $x(k)$, comprising the A/C angular coordinate and ancillary variables, given the sequence of observations $\{\mathcal{Y}(k),\ k=1,2,\ldots\}$, where, for the kth scan,

$$\mathcal{Y}(k) \triangleq \{y(t_k,\tau),\ 0 < \tau < T\}. \qquad \text{(II-1)}$$

Such algorithms generically are characterized by the following functions:

<u>Extrapolation</u>: $\hat{x}(k|k-1) = f(\hat{x}(k-1|k-1),\ k,\ k-1)$ (II-2)

<u>Error Estimation</u>: $\hat{e}(k|k) = g(\mathcal{Y}(k),\ \hat{\mathcal{Y}}(k|k-1))$ (II-3)

<u>Updating</u>: $\hat{x}(k|k) = \hat{x}(k|k-1) + \hat{e}(k|k)$ (II-4)

where

$\hat{x}(k-1|k-1) \triangleq$ an estimate of $x(k-1)$, given all observations up through the (k-1)th scan. (II-5)

$\hat{x}(k|k-1) \triangleq$ the extrapolation of $\hat{x}(k-1|k-1)$ up to the beginning of the kth scan (II-6)

$\hat{e}(k|k) \triangleq$ an estimate of the error in $\hat{x}(k|k-1)$, given the kth scan observations $\mathcal{Y}(k)$ (II-7)

$\hat{\mathcal{Y}}(k|k-1) \triangleq$ prediction of $\mathcal{Y}(k)$, based on the extrapolated estimate $\hat{x}(k|k-1)$. (II-8)

The dependency of $\hat{e}(k|k)$ on $\hat{x}(k|k-1)$ through $\hat{\mathcal{Y}}(k|k-1)$ indicates that such algorithms are recursive. The theory and design of recursive state estimators is well-documented, and <u>given suitable, valid models</u>, this approach might be applied to the extended problem involving also identification of several model parameters imprecisely known. Modeling the state excitation as white noise and augmentation of the state with the parameters to be identified are required generally. Recursive estimation has much to

recommend it generally, but the potentially high dimensionality of the augmented state model and reservations about the validity in this application of modeling state evolution uncertainty with a white state noise both have made more attractive a layered approach to the extended problem, as follows:

1. Use modified recursive estimation of the angular coordinate and ancillary variables (simple state).
2. Use batch (i.e. finite-memory) processing of a sequence of most recent, raw state estimates to obtain a more refined state evolution model.
3. Extrapolate with the most recent refinement of the state evolution model.

Modified recursive estimation refers to the error estimate $\hat{e}$ being defined and calculated for its direct addition with constant unity gain in the update equation. This is necessary to insure that the results of the batch processing include the most recent observations, fully weighted. In this way also both tracking stability (including false lock) and estimation quality (including suppression of multipath effects) can be dealt with in developing the estimate $\hat{e}$. For example, clearly $\hat{e}(k|k)$ is a function of the actual error $e(k)$ in $\hat{x}(k|k-1)$, where

$$e(k) \triangleq x(k) - \hat{x}(k|k-1). \qquad \text{(II-9)}$$

Strict stability of the tracking algorithm requires

$$\langle\hat{e}\rangle = 0 \quad \text{when } e = 0 \qquad \text{(II-10)}$$

$$0 < \langle\hat{e}\rangle < 2e \quad \text{when } e > 0 \qquad \text{(II-11)}$$

$$2e < \langle\hat{e}\rangle < 0 \quad \text{when } e < 0. \qquad \text{(II-12)}$$

Estimation criteria for e should optimally relate to the quality of the x estimates, perhaps, but with the assumption that x is a fixed parameter throughout the kth scan observations $\mathcal{Y}(k)$, a reasonable, albeit possibly suboptimal, approach is to require $\hat{e}$ to be locally unbiased at $e = 0$ and have minimum mean square error, i.e.

Locally Unbiased: $\langle\hat{e} - e\rangle\Big|_{e=0} = 0$ (vector) (II-13)

$$\left[\frac{\partial}{\partial e}\langle\hat{e} - e\rangle\right]_{e=0} = 0 \text{ (matrix)} \qquad \text{(II-14)}$$

MMSE: $\Phi_0^{-1} \stackrel{\Delta}{=} \langle(e - e)(e - e)^T\rangle\Big|_{e=0}$ be minimum among all estimates of e locally unbiased at e=0 (II-15)

This is the locally optimum estimation criterion expounded by Murphy in [2] and applied in [1] and in the next chapter of this report to estimation of the A/C angular coordinate, given respectively the (linear) i-f signal and then its envelope. Approximate covariance-of-error expressions for the estimate $\hat{e}$ are produced also in terms of the estimated signal-to-noise ratio on the kth scan; these might be useful for weighting the raw estimates $\hat{x}$ in the batch processing, as shown below.

Batch processing, corresponding to finite-memory or moving-window filtering, is used here to fit in a least squares sense on assumed state evolution law linear in the unknown parameters, constituting a vector V, to a sequence of estimates $\hat{x}$, constituting a vector X, via minimization of the quadratic form

$$(HV - X)^T\Psi(HV - X)\Big|_{\Psi=W^TW} = (WHV - WX)^T(WHV - WX) \qquad \text{(II-16)}$$

giving

$$\hat{V} = (WH)^+WX = \hat{V}(k) \qquad \text{(II-17)}$$

where $(\ )^+$ denotes the pseudoinverse of the matrix ( ). If a linear law of evolution is assumed of the form

$$x(t) = x(t_k) + (t - t_k)\dot{x}(t_k) \qquad \text{(II-18)}$$

and the most recent K + 1 raw estimates $\hat{x}$ and associated error variances σ_e^2 are used, then

$$X = \begin{pmatrix} \hat{x}(k|k) \\ \hat{x}(k-1|k-1) \\ \vdots \\ \hat{x}(k-K|k-K) \end{pmatrix} \tag{II-19}$$

$$H = \begin{pmatrix} 1 & 0 \\ 1 & (t_{k-1}-t_k) \\ \vdots & \\ 1 & (t_{k-K}-t_k) \end{pmatrix} \tag{II-20}$$

and

$$W = \begin{pmatrix} \frac{1}{\sigma_e(k)} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_e(k-1)} & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \frac{1}{\sigma_e(k-K)} \end{pmatrix} \tag{II-21}$$

Under these circumstances H and WH are K + 1 x 2 with full rank 2 and thus as an alternative to the expression (II-17) for $\hat{V}$ above, we have

$$\hat{V} = (H^T \Psi H)^{-1} H \Psi X = \hat{V}(k) \tag{II-22}$$

In all cases the matrices to be inverted are either 2 × 2 in dimension or otherwise diagonal (approximately), so the computational load for model estimation is not excessive. The number of measurements $K(\geq 2)$ used here needs to be established; this might be done in a manner that would make the algorithm somewhat adaptable to A/C maneuvering. This will be studied.

Given a solution for $\hat{V}(x)$, extrapolation for $\hat{x}(k+1|k)$ is accomplished as follows:

$$\hat{x}(k+1|k) = (1 \quad (t_{k+1}-t_k))\hat{V}(k) \qquad \text{(II-23)}$$

We note this is a smoothed prediction; the smoothed "current estimate" value is available as the first element of $\hat{V}(k)$. The one-step prediction is expected to be the principal result, however, both for maintenance of the algorithm (in calculating $\hat{e}$) and in output, since processing time is offset somewhat in prediction.

In summary, a layered tracking algorithm structure has been described involving recursive estimation of the A/C coordinate and batch processing of these estimates for model identification. The approach allows false-lock prevention and suppression of local multipath effects (assymetrical pulse distortion) to be included in the well-defined problem of processing new observations for optimal estimation of prediction error. Also the approach permits the batch processing window to be adaptable to manifest A/C dynamics, thus producing a good and reasonably recent model for extrapolating the estimate through gross multipath effects, such as signal fades. A disadvantage of batch processing is the storage requirement for past data, if significant. An overall study of processing time, storage requirements and tracking performance is being done. Fully-recursive approaches have not been totally discounted either and one also being studied. Particularly attractive are the two-filter versions of Bierman [3] for fixed memory filtering and Nelson and Stear [4] for simultaneous state and parameter estimation. Batch processing and recursive estimation approaches may not produce the same results [5], and performance analyses and comparisons are essential, along with assessments of computational loads, in selecting the algorithm to be implemented for field test.

III. LOCALLY OPTIMUM PREDICTION ERROR ESTIMATION
Part A: BASIC THEORY AND I-F SIGNAL PROCESSING

The concept and development of locally optimum estimation, expounded by Murphy [2] in 1968, was summarized in [1] and applied to the A/C angular coordinate estimation problem in MLS, given the receiver i-f signal. To facilitate presentation and evaluation of new results to be given, a brief summary of some results from [1] is presented in this chapter including key relations in the locally optimum estimation model.

Locally optimum estimation theory involves operations on the Radon-Nikodym derivative dP_e/dQ where P_e is the measure corresponding to the integrated observations process $\{Y(t),\ t \in [0,T]\}$, and Q is the measure corresponding to the integrated receiver noise process $\{N(t),\ t \in [0,T]\}$; P_e is absolutely-continuous with respect to (wrt) Q. The Radon-Nikodym derivative is a generalization of the ratio of the two appropriate probability densities, commonly termed the likelihood ratio λ, to which it degenerates when the measures P_e and Q are both absolutely-continuous wrt Lebesgue measure μ. Without regard for these finer distinctions, the symbol λ and term likelihood ratio will be used indiscriminantly for either mathematical object, as applicable, thereby rendering the basic model applicable to both continuous-time and discrete-time estimation, the latter being used in the envelope processing algorithm where probability densities of the (finite) sets of samples are available. Specifically, therefore

$$\text{Likelihood Ratio, } \lambda_e = \begin{cases} \dfrac{dP_e}{dQ}, & \text{for continuous-time processes with measures } P_e, Q \text{ as defined.} \\[2ex] \dfrac{p(V)|_{\text{signal present}}}{p(V)|_{\text{signal absent}}}, & \text{for discrete-time processes, } V \text{ being the K-vector of envelope samples, } [V(t_k,\tau_1), \ldots V(t_k,\tau_K)]^T. \end{cases} \tag{III-1}$$

Murphy [2] has shown that the estimate $\hat{e}$ of the vector e which is locally optimum at e = 0 is given by

$$\hat{e} = \Phi_0^{-1}\Lambda_0 \tag{III-2}$$

where Λ_0 is the vector whose ith component, Λ_{0_i}, is given by

$$\Lambda_{0_i} = \begin{cases} \left[\frac{\partial}{\partial e_i} \ln \lambda_e\right]_{e=0} , & \text{if } \lambda_e \neq 0 \\ 0 & , \text{otherwise} \end{cases} \tag{III-3}$$

and

$$\Phi_0 = \langle \Lambda_0 \Lambda_0^T \rangle \;, \tag{III-4}$$

the latter expectation being taken wrt the measure P_o corresponding to $e = 0$. Further, if $e = 0$, the residual mean square error (i.e., the error covariance matrix associated with this estimate is Φ_0^{-1} , that is

$$\langle \hat{e}\hat{e}^T \rangle \big|_{e=0} = \Phi_0^{-1} \;, \tag{III-5}$$

Applying these results to estimation of the error in the one-step prediction $\hat{x}(k|k-1)$, given the kth scan observation $Y(k)$, the error vector e was constituted as follows:

$$e = \begin{pmatrix} e_\theta \\ e_\alpha \end{pmatrix} = \begin{pmatrix} \text{error in the prediction, } \hat{\theta}(t_k|t_{k-1}) \\ \text{error in the prediction, } \hat{\alpha}(t_k|t_{k-1}) \end{pmatrix} \tag{III-6}$$

corresponding to the state vector x being estimated. Four cases have been or are under consideration at this time, as follows:

1. I-F signal observations
   a. No multipath
   b. With multipath
2. Envelope observations
   a. No multipath
   b. With multipath

These studies and results are described below and in the following chapter.

1. I-F Signal Observations

The i-f signal $y(t_k,\tau)$ was given in Chapter I as follows:

$$y(t_k,\tau) = y_D(t_k,\tau) + y_R(t_k,\tau) + n(\tau) \qquad \text{(III-7)}$$

where, for $0 < \tau < T$ on the kth scan, neglecting doppler effects,

$$y_D(t_k,\tau) = \alpha(t_k)p[\theta_A(\tau) - \theta(t_k)]\cos[\omega_{IF}\tau + \beta(t_k)] \qquad \text{(III-8)}$$

$$y_R(t_k,\tau) = \sum_i \alpha_i(t_k,\tau)p[\theta_A(\tau - \frac{\Delta r_i(t_k)}{c}) - \theta_{R_i}(t_k)]\cos[\omega_{IF}\tau + \beta_i(t_k)] \qquad \text{(III-9)}$$

and, since the i-f bandwidth B($\simeq$ 160 kHz) is substantially greater than the reciprocal pulse width (8 millisec^{-1}),

$$n(t) \simeq \text{white Gaussian noise with 2-sided spectral density } N_o (\simeq \sigma_n^2/2B). \qquad \text{(III-10)}$$

In the following $\hat{y}_D$ denotes the above expression for y_D with the estimates $\hat{\alpha}(t_k|t_{k-1})$, $\hat{\theta}(t_k|t_{k-1})$ substituted for α, θ respectively; also the explicit dependency on t_k is suppressed when no confusion results.

a. No Multipath. Here $y_R \equiv 0$; under the hypothetical assumption that the parameter β is known, the likelihood ratio, λ, for instrumentation purposes, can be written as

$$\lambda_e = \exp\{\frac{1}{N_o}\int_0^T y_D(\tau)[y(\tau) - \tfrac{1}{2}y_D(\tau)]d\tau\} \qquad \text{(III-11)}$$

Also, under the same circumstances,

$$\Lambda_0 = \frac{1}{N_o}\begin{pmatrix} \int_0^T \frac{\partial\hat{y}_D(\tau)}{\partial\hat{\theta}}[y(\tau) - \hat{y}_D(\tau)]d\tau \\ \int_0^T \frac{\partial\hat{y}_D(\tau)}{\partial\hat{\alpha}}[y(\tau) - \hat{y}_D(\tau)]d\tau \end{pmatrix} \qquad \text{(III-12)}$$

where $\hat{\theta}$, $\hat{\alpha}$ denote $\hat{\theta}(t_k|t_{k-1})$, $\hat{\alpha}(t_k|t_{k-1})$ respectively. Finally, and most importantly, after substituting for y_D (III-8), and some manipulation and simplification, we obtain Φ_0^{-1}, the covariance of the estimation error

$$\varepsilon \triangleq \hat{e} - e \tag{III-13}$$

for e = 0, as follows:

$$\Phi_0^{-1} \approx 2N_o \begin{pmatrix} \{\hat{\alpha}^2 \int_0^T \dot{p}^2[\theta_A(\tau)]d\tau\}^{-1} & 0 \\ 0 & \{\int_0^T p^2[\theta_A(\tau)]d\tau\}^{-1} \end{pmatrix} \tag{III-14}$$

where

$$\dot{p}[\theta_A(\tau)] \triangleq \frac{d}{d\theta_e} p(\theta_e)\Big|_{\theta_e=\theta_A(\tau)}. \tag{III-15}$$

The case for β unknown but assumed to be a random variable uniformly distributed on $(-\pi,\pi)$ will not be summarized here; reference is made to [1] for this development and the resulting structure, which is a quite complicated quadrature detector signal processor.

b. With Multipath. The processor of the signal for the no-multipath case above, as evidenced in the expressions for Λ_0 above and in [1], consists of an integration over the scan interval of the product of the received i-f signal and a sinusoid which is both phase-synced with the i-f signal and amplitude-modulated by the derivative (wrt the parameter of interest) of the direct path envelope function. This is a form of gating (weighting), of course, and in reaching for a concept of multipath-adaptive reception, it seemed reasonable to expect the same general structure in the nucleus of an adaptive receiver with the amplitude-modulation of the local oscillator being controlled by suitable additional algorithmic machinery to adaptively discriminate against manifest multipath interference. These notions are fairly easily confirmed for the i-f signal observations case, using the locally

optimum estimation equations.

The multipath propagation component $y_R(t_k,\tau)$ of the composite i-f signal $y(t_k,\tau)$ is modeled as the sum of individual reflection components $y_{R_i}(t_k,\tau)$, each a function of an amplitude variable $\alpha_i(t_k,\tau)$ and a phase variable $\beta_i(t_k)$. Within a given scan period the β_i have been effectively determined to be independent random variables uniformly distributed on $[-\pi,\pi]$ (through their individual variations from scan-to-scan are highly correlated, thus insuring the continuity in time of the interference phenomenon present). Under the assumption each $\alpha_i(t_k,\tau)$ is a (time-varying) Rayleigh random variable (i.e., a Rayleigh random process) independent of the associated $\beta_i(t_k)$, each reflection component $y_{R_i}(t_k,\tau)$ is a nonstationary Gaussian process, as is consequently also the total multipath propagation component $y_R(t_k,\tau)$. Under milder assumptions on the $\alpha_i(t_k,\tau)$, if the number of non-zero α_i's are large, it is still possible to argue that $y_R(t_k,\tau)$ is approximately Gaussian. It seems plausible that usually some combination of the preceeding conditions would prevail, such that the multipath interference component $y_R(t_k,\tau)$ in the received i-f signal $y(t_k,\tau)$ is an additive Gaussian random process with zero mean value, or more appropriately a random field with zero mean value and covariance

$$\left\langle y_R(t_i,\tau_j)y_R(t_k,\tau_\ell)\right\rangle = \phi_R(t_i,t_k,\tau_j,\tau_\ell) \tag{III-16}$$

This assumption we make.

Let the composite i-f signal $y(t_k,\tau)$ be sampled on the kth scan and a K-vector $Y(t_k)$ be defined, comprising these samples, i.e.,

$$Y(t_k) = \begin{pmatrix} y(t_k,\tau_1) \\ y(t_k,\tau_2) \\ \vdots \\ y(t_k,\tau_K) \end{pmatrix} = \begin{pmatrix} y_D(t_k,\tau_1) \\ y_D(t_k,\tau_2) \\ \vdots \\ y_D(t_k,\tau_K) \end{pmatrix} + \begin{pmatrix} y_R(t_k,\tau_1) \\ y_R(t_k,\tau_2) \\ \vdots \\ y_R(t_k,\tau_K) \end{pmatrix} + \begin{pmatrix} n(\tau_1) \\ n(\tau_2) \\ \vdots \\ n(\tau_K) \end{pmatrix} \tag{III-17}$$

$$= Y_D(t_k) + [Y_R(t_k) + N], \tag{III-18}$$

defining the K-vectors Y_D, Y_R, and N. Regarding Y_D as the signal (known sure function) and Y_R + N as the interference (Gaussian), then the likelihood ratio, corresponding to (III-16) above, is

$$\lambda = \frac{p_{Y_R+N}(Y-Y_D)}{p_{Y_R+N}(Y)} = \exp\{-\tfrac{1}{2}[(Y-Y_D)^T(\Phi_R+\Phi_N)^{-1}(Y-Y_D) - Y^T(\Phi_R+\Phi_N)^{-1}Y]\} \quad \text{(III-19)}$$

where Φ_N is the covariance of the noise vector N, and $\Phi_R(t_k)$ is the covariance of the reflection component vector $Y_R(t_k)$. (The ijth entry in $\Phi_R(t_k)$ is $\phi_R(t_k,t_k,\tau_i,\tau_j)$, from (III-16).) The above can be simplified to

$$\lambda = \exp[Y_D^T(\Phi_R + \Phi_N)^{-1}(Y - \tfrac{1}{2}Y_D)], \quad \text{(III-20)}$$

corresponding to equation (III-11) above. Then, corresponding to (III-12), the optimal processing of the received signal $Y(t_k)$ is indicated by

$$\Lambda_0 = \begin{pmatrix} \dfrac{\partial \hat{Y}_D^T}{\partial \hat{\theta}} (\Phi_R + \Phi_N)^{-1}(Y - \hat{Y}_D) \\ \dfrac{\partial \hat{Y}_D^T}{\partial \partial} (\Phi_R + \Phi_N)^{-1}(Y - \hat{Y}_D) \end{pmatrix} \quad \text{(III-21)}$$

which, if the i-f noise samples are independent with variance σ_n^2, then $\Phi_N = \sigma_n^2 I$, and Λ_0 can be expressed in the form

$$\Lambda_0 = \frac{1}{\sigma_n^2} \begin{pmatrix} \dfrac{\partial \hat{Y}_D^T}{\partial \hat{\theta}} [I - \dfrac{\Phi_R}{\sigma_n^2}(I + \dfrac{\Phi_R}{\sigma_n^2})^{-1}](Y - \hat{Y}_D) \\ \\ \dfrac{\partial \hat{Y}_D^T}{\partial \hat{\alpha}} [I - \dfrac{\Phi_R}{\sigma_n^2}(I + \dfrac{\Phi_R}{\sigma_n^2})^{-1}](Y - \hat{Y}_D) \end{pmatrix} \quad \text{(III-22)}$$

The innovations $(Y - \hat{Y}_D)$ are effectively processed with a weighting function that is dependent on Φ_R, i.e., $\frac{\partial \hat{Y}_D^T}{\partial \hat{\theta}} \left[I - \frac{\Phi_R}{\sigma_n^2} \left(I + \frac{\Phi_R}{\sigma_n^2} \right)^{-1} \right]$, in the angular coordinate channel. When there is no multipath and hence $\Phi_R = 0$, the results correspond with equation (III-12) above, otherwise a modification dependent on Φ_R is made that preserves locally optimum estimation performance. Application of the algorithm requires estimation of Φ_R, of course, as well as Φ_N; however as (III-21) shows, only the sum $(\Phi_R + \Phi_N)$ is needed not the individual covariances. The sum $(\Phi_R + \Phi_N)$ is the covariance of the innovations process $(Y - \hat{Y}_D)$ at $e = 0$, however, suggesting a method by which $\Phi_R + \Phi_N$ might be determined and even tracked as Φ_R varies with the environment.

No further study of i-f signal processors in general, and their multipath suppression capabilities in particular, has been done, or is planned in view of the competitive performance observed of the much more economical envelope processors, but the notion of interference-adaptive reception is clearly capable of generalization to non-additive, non-Gaussian interference models. This work is in progress.

IV. LOCALLY OPTIMUM PREDICTION ERROR ESTIMATION

Part B: ENVELOPE SIGNAL PROCESSING AND PERFORMANCE EVALUATIONS

As pointed out in the December 1975 interim report [1], the complexity of the optimal i-f signal processors motivates serious consideration of the simple envelope detector and the optimal processing of the resulting envelope samples in the estimation of A/C angular coordinates. To date the envelope processor has been developed only for the no multipath case and an outline of this development is presented below.

Murphy [2] (see also Chapter III of [1]) has shown that the locally optimum estimate $\hat{e}$ of the parameter vector e at e = 0 is given by

$$\hat{e} = \Phi_0^{-1}\Lambda_0 \tag{IV-1}$$

where Λ_0 is the vector whose ith component is

$$\Lambda_{0_i} = \begin{cases} \frac{\partial}{\partial e_i} \ln\left(\frac{dP_e}{dQ}\right)\Big|_{e=0} , & \text{if } \frac{dP_e}{dQ} \neq 0 \\ 0 , & \text{otherwise} \end{cases} \tag{IV-2}$$

and

$$\Phi_0 = \langle\Lambda_0\Lambda_0^T\rangle = \int_\Omega \Lambda_0\Lambda_0^T dP_0 \tag{IV-3}$$

It is assumed that, for the samples of the envelope, the Radon-Nikodym derivative is equal to the likelihood ratio, i.e.,

$$\frac{dP_e}{dQ}\Bigg|_{e=0} = \frac{p(V|S+N)}{p(V|N)}\Bigg|_{e=0} \tag{IV-4a}$$

$$= \prod_{j=1}^{K} I_0\left(\frac{V_jP_j}{\sigma_n^2}\right)\exp\left(\frac{-P_j^2}{2\sigma_n^2}\right) \tag{IV-4b}$$

which further assumes the i-f noise is a narrow band, zero mean GRP with variance σ_n^2 and that the envelope detector function is given by equation (I-3) with $y_{R_s} = y_{R_c} = 0$. These latter assumptions lead to the determination that the probability density function for the envelope samples given that e = 0 is [6, p. 166 and p. 357]

$$p(V_j|e=0) = \frac{V_j}{\sigma_n^2} I_0\left(\frac{V_j P_j}{\sigma_n^2}\right)\exp - \left(\frac{V_j^2 + P_j^2}{2\sigma_n^2}\right), \quad V_k \geq 0 \tag{IV-5}$$

where $\{V_j\}$ is a set of identically distributed (Rician) random variables. Note that $V_j \triangleq V(t_k,\tau_j)$ and $P_j \triangleq P(t_k,\tau_j) = \alpha(t_k)p[\theta_A(\tau_j) - \theta(t_k)] = \alpha p_j$ are the sample values of the envelope and the deterministic signal due to the antenna selectivity function, respectively, at $\tau = \tau_j$ on the kth scan. Also recall [1] that e is the error vector in the parameters being estimated, and in the present case

$$e = \begin{pmatrix} e_\theta \\ e_\alpha \end{pmatrix} \tag{IV-6}$$

where $e_\theta = \theta - \hat{\theta}$ and $e_\alpha = \alpha - \hat{\alpha}$, i.e., the errors in estimating the angle θ and the amplitude of the direct path signal α; and therefore e = 0 implies $\theta = \hat{\theta}$ and $\alpha = \hat{\alpha}$ (not $\theta = \alpha = 0$). It should also be noted at this point that σ_n^2 must be assumed known (or being provided via a separate estimation process).

The form of the likelihood ratio given in (IV-4b) is obtained by assuming independence of the K envelope samples which implies $p(V_1, \ldots V_k) = \prod_{j=1}^{K} p(v_j)$ and dividing by the product of Rayleigh densities $p(V_j|P_j=0)$. The independence assumption implies that the sampling interval $\Delta\tau = \tau_j - \tau_{j-1}$ is equal to the value of the delay variable which makes the autocorrelation function of the i-f noise equal zero [7, p. 399 and p. 416]. If the i-f noise spectra is assumed to be ideal bandpass, then $\Delta t = (\text{i-f Bandwidth})^{-1} \approx 6\mu s$.

The components of Λ_0 from (IV-2) are the partial derivatives of the log likelihood ratio ℓ_k and are given by

$$q_1 \triangleq \frac{\partial \ell_k}{\partial \theta}\bigg|_{e=0} = \frac{1}{\sigma_n^2} \sum_{j=1}^{K} \alpha \frac{\partial p_j}{\partial \theta} \{V_j \frac{I_1(\frac{V_j P_j}{\sigma_n^2})}{I_0(\frac{V_j P_j}{\sigma_n^2})} - P_j\}_{e=0} \tag{IV-7}$$

$$q_2 \triangleq \frac{\partial \ell_k}{\partial \alpha}\bigg|_{e=0} = \frac{1}{\sigma_n^2} \sum_{j=1}^{K} p_j \{V_j \frac{I_1(\frac{V_j P_j}{\sigma_n^2})}{I_0(\frac{V_j P_j}{\sigma_n^2})} - P_j\}_{e=0} \tag{IV-8}$$

(Note that $q_1 = q_2 = 0$ are the maximum likelihood equations for $\hat{e}_{ML}$, i.e., $\theta = \hat{\theta}_{ML}$ and $\alpha = \hat{\alpha}_{ML}$.) Thus

$$\Lambda_0 \Lambda_0^T = \begin{pmatrix} q_1^2 & q_1 q_2 \\ q_2 q_1 & q_2^2 \end{pmatrix} \tag{IV-9}$$

and

$$\Phi_0 = \langle \Lambda_0 \Lambda_0^T \rangle = \begin{pmatrix} \langle q_1^2 \rangle & \langle q_1 q_2 \rangle \\ \langle q_2 q_1 \rangle & \langle q_2^2 \rangle \end{pmatrix} \tag{IV-10}$$

As the locally optimum estimator $\hat{e}$ is unbiased at $e = 0$

$$\langle \hat{e} \rangle = \Phi^{-1} \langle \Lambda_0 \rangle = \Phi^{-1} \begin{pmatrix} \langle q_1 \rangle \\ \langle q_2 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{IV-11}$$

To show that this is indeed the case, it suffices to show, using freely some notational abuse,

$$\left\langle V_j \frac{I_1\left(\frac{V_j P_j}{\sigma_n^2}\right)}{I_0\left(\frac{V_j P_j}{\sigma_n^2}\right)} \right\rangle = P_j \tag{IV-12}$$

$$= \int_0^\infty V_j \frac{I_1(\ )}{I_0(\ )} \{\frac{V_j}{\sigma_n^2} I_0(\ )\exp - (\frac{V_j^2 + P_j^2}{2\sigma_n^2})\ dV_j$$

$$= \int_0^\infty \frac{V_j^2}{\sigma_n^2} I_1(\ )\exp - (\frac{V_j^2 + P_j^2}{2\sigma_n^2})\ dV_j$$

$$= P_j Q_2(x,0) = P_j$$

where

$$Q_m(x,y) \triangleq \int_y^\infty Z(\frac{Z}{x})^{m-1} I_{m-1}(xZ)\exp - (\frac{Z^2 + x^2}{2})\ dZ \tag{IV-13}$$

is the generalized Marcum Q function [8, p. 411] and $Q_m(x,0) = 1$, all m and x (the integrand is the probability density function of the random variable $Z\sigma = ||Y||$ where $Y \in N_{2m}(A,\sigma^2)$ and $x\sigma = ||A||$, see [9, pp. 41-42]).

Now the expressions for q_1 and q_2 (equations (IV-7) and (IV-8)) may be simplified by letting

$$y_j \triangleq V_j \frac{I_1(\ )}{I_0(\ )} - P_j \tag{IV-14}$$

be a new random variable with $\langle y_j \rangle = 0$ and $\langle y_i y_j \rangle = 0$ for $i \neq j$. Thus (IV-7) and (IV-8) may now be written as

$$q_1 = \frac{1}{\sigma_n^2} \sum_{j=1}^{K} \alpha \dot{p}_j y_j \tag{IV-15}$$

$$q_2 = \frac{1}{\sigma_n^2} \sum_{j=1}^{K} p_j y_j \tag{IV-16}$$

with the obvious notation $\dot{p}_j = \frac{\partial p_j}{\partial \theta}$. The development of the locally optimum estimation algorithm will be complete with the evaluation of the components in the Φ_0 matrix of equation (IV-10) and the consequent inversion to provide Φ^{-1}. Therefore, proceeding to evaluate the entries in (IV-10), yields

$$\langle q_1^2 \rangle = \frac{1}{\sigma_n^4} \sum_{j=1}^{K} \alpha^2 \dot{p}_j^2 \ \langle y_j^2 \rangle \tag{IV-17a}$$

$$\langle q_2^2 \rangle = \frac{1}{\sigma_n^4} \sum_{j=1}^{K} p_j^2 \ \langle y_j^2 \rangle \tag{IV-17b}$$

$$\langle q_1 q_2 \rangle = \langle q_2 q_1 \rangle = \frac{1}{\sigma_n^4} \sum_{j=1}^{K} \alpha p_j \dot{p}_j \ \langle y_j^2 \rangle \tag{IV-17c}$$

Note that (IV-17c) implies that if p_j has even symmetry and $\dot{p}_j$ has odd symmetry about the mid-point of the summation, then $\langle q_1 q_2 \rangle = 0$ ($\langle y_j^2 \rangle$ has even symmetry if p_j does). The conditions required for this to be true are:

1. the antenna selectivity function $p(e_\theta)$ has even symmetry about its boresight,
2. the derivative of $p(e_\theta)$ with respect to the angle off boresight has odd symmetry about its boresight, and
3. the sampling times τ_k are symmetrically distributed about the center (boresight) of the stored signal $p(e_\theta)$.

The only significant problem remaining is the evaluation of $\langle y_j^2 \rangle$. From the defining equation for y_j, (IV-14), it is clear that

$$\langle y_j^2 \rangle = \langle [V_j \frac{I_1(\ )}{I_0(\ )}]^2 \rangle - P_j^2 \tag{IV-18}$$

where

$$\left\langle \left[V_j \frac{I_1(\ )}{I_0(\ )}\right]^2 \right\rangle = \int_0^\infty \left[V_j \frac{I_1(\ )}{I_0(\ )}\right]^2 \frac{V_j}{\sigma_n^2} I_0(\ ) \exp - \left(\frac{V_j^2 + P_j^2}{2\sigma_n^2}\right) dV_j \qquad \text{(IV-19)}$$

Due to the nonlinear nature of $\frac{I_1(\ )}{I_0(\ )}$, where $(\ ) = \left(\frac{V_j P_j}{\sigma_n^2}\right)$, this integral has not yet yielded to an exact evaluation. Certainly this integral could be evaluated via careful digital computation; however, the value of the integral is a function of the (signal-to-noise) ratio $R_j^2 = \frac{P_j^2}{2\sigma_n^2} = \frac{\alpha^2 p_j^2}{2\sigma_n^2} \triangleq R_{max}^2 p_j^2$ and would, therefore, require on line computation for each sample value p_j and scan estimate $\hat{R}_{max} = \frac{\hat{\alpha}^2}{2\hat{\sigma}_n^2}$. As this appears to be an unreasonable computational burden on the anticipated receiver microprocessor, analytic approximations were investigated. In order to minimize the assumptions made, power series approximations to the $\frac{I_1(\ )}{I_0(\ )}$ function were used but the resulting integrals, which were put in the form of the moments of the probability density function associated with $Q_2(x,y)$, are given in terms of the confluent hypergeometric function which is expressed either as an infinite sum or in terms of exponentials and I_1 and I_0. In the first case the value of the integral is expressed as an infinite sum of an infinite sum, and in the second as an infinite sum of products of exponentials and modified Bessel functions. In neither case were closed form solutions evident; nor was it clear how reasonable assumptions might simplify the expressions.

Approximations to the values of the integrals can be accomplished by examining the nature of the nonlinearity and the parameterization of the Rician density function, i.e., its functional character for high and low values of R_j^2, e.g. [7, p. 414] Gaussian for large R_j^2 and Rayleigh for small R_j^2 (= 0). That this is a reasonable approach is supported by the fact that the antenna selectivity function p_j is, by design, a highly selective function, i.e., its values tend to be relatively high in the mainbeam and fall off rapidly to relatively low values in the side lobes, e.g.-23db. Thus it seems reasonable to dichotomize the problem into these two extremes of R_j^2 and use asymptotic approximations in these two cases.

Case I: $R_j^2 >> 1$

Because the final result desired is the evaluations of $<y_j^2>$ as given in (IV-18) which is the difference in two relatively large numbers which may differ only slightly for $R_j^2 >> 1$, care must be taken in approximating the value of the integral in (IV-19). For example, assuming $\frac{I_1(\ )}{I_0(\ )} \approx 1$ for $(\ ) = \frac{V_j P_j}{\sigma_n^2} >> 1$ and assuming the Rician density becomes Gaussian with mean P_j and variance σ_n^2 yields $<V_j^2> = P_j^2 + \sigma_n^2$ with $<y_j^2> = \sigma_n^2$. Making the same assumption that $\frac{I_1(\ )}{I_0(\ )} = 1$ but retaining the exact form of the Rician density yields [7, p. 415] $<V_j^2> = P_j^2 + 2\sigma_n^2$ with $<y_j^2> = 2\sigma_n^2$, a variation of 2-to-1 from the previous result. The following analysis produces an upper bound on the value of the integral which appears to be reasonably accurate.

To avoid squaring the approximation of the $\frac{I_1(\ )}{I_0(\ )}$ function in the integrand, use, dropping the subscripts,

$$[V \frac{I_1(\ )}{I_0(\ )}]^2 I_0(\ ) = [V \frac{I_1(\ )}{I_0(\ )}] I_1(\ ) \cdot V \tag{IV-20}$$

with (IV-19) now becoming

$$\left\langle [V \frac{I_1(\ )}{I_0(\ )}]^2 \right\rangle = \int_0^\infty [V \frac{I_1(\ )}{I_0(\ )}] \frac{V^2 P}{\sigma_n^2 P} I_1(\ ) \exp - (\frac{V^2 + P^2}{2\sigma_n^2}) \, dV \tag{IV-21}$$

$$= P \left\langle V \frac{I_1(\ )}{I_0(\ )} \right\rangle, \text{ expectation wrt } N_4(P,\sigma^2) \text{ [9, p. 42]}$$

Now introducing the approximation $\frac{I_1(\ )}{I_0(\ )} \approx 1 (\geq \frac{I_1(\ )}{I_0(\ )})$

$$P \left\langle V \frac{I_1(\ )}{I_0(\ )} \right\rangle_{N_4} \leq P < V >_{N_4} \tag{IV-22}$$

$$\leq P(2\sigma_n^2)^{\frac{1}{2}} e^{-R^2} \Gamma(2.5) {}_1F_1(2.5;2;R^2)$$

$$\leq \frac{(2\pi)^{\frac{1}{2}}}{4} P\sigma_n e^{-R^2/2} I_0(R^2/2)[3 + \frac{I_1(R^2/2)}{I_0(R^2/2)} + 2R (1 + \frac{I_1(R^2/2)}{I_0(R^2/2)})]$$

This upper bound is used in obtaining the upper bound on the ratio $\lambda = \langle y_j^2 \rangle / \sigma_n^2$ in Table (IV-1). This upper bound calculation, in addition to several digital computations of the values of the integral of (IV-19), leads to the choice $\lambda = 1$, i.e., $\underline{\langle y_j^2 \rangle \simeq \sigma_n^2 \text{ for } R^2 > 1}$.

Case II: $R_j^2 < 1$

Using the same approach as above, except approximating $\frac{I_1(x)}{I_0(x)}$ by $\frac{x}{2}$ for $R^2 < 1$, yields

$$P \left\langle V \frac{I_1(\ )}{I_0(\ )} \right\rangle_{N_4} \leq P \left\langle \frac{V^2 P}{2\sigma_n^2} \right\rangle_{N_4} = \frac{P^2}{2\sigma_n^2} \left\langle V^2 \right\rangle_{N_4} \tag{IV-23}$$

$$\leq \left(\frac{P^2}{2\sigma_n^2}\right) 2\sigma_n^2 e^{-R^2} \Gamma(3)\, {}_1F_1(3;2;R^2)$$

$$\leq 2P^2\left(1 + \frac{R^2}{2}\right)$$

Using this upper bound for $R^2 < 1$ yields

$$\langle y_j^2 \rangle \simeq P_j^2 (1 + R_j^2) \tag{IV-24}$$

To use these approximations in (IV-17) assume that there are $\frac{K}{2}$ samples taken in each of the to-and fro-scans (K/scan) and that $R_j^2 < 1$ for $1 \leq j \leq \ell$ and $R_j^2 \geq 1$ for $\ell+1 \leq j \leq K$. Therefore

$$\langle q_1^2 \rangle \simeq \frac{\alpha^2}{\sigma_n^4} \left\{ \sum_{j=1}^{\ell} \dot{\rho}_j^2 P_j^2 (1 + R_j^2) + \sum_{j=\ell+1}^{K} \dot{\rho}_j^2 \sigma_n^2 \right\} \tag{IV-25a}$$

$$\simeq \frac{\alpha^2}{\sigma_n^2} \left\{ 2 \sum_{j=1}^{\ell} \dot{\rho}_j^2 R_j^2 (1 + R_j^2) + \sum_{j=\ell+1}^{K} \dot{\rho}_j^2 \right\} \tag{IV-25b}$$

and for $R_j^2 \ll 1$ for $1 \leq j \leq \ell$

$$\langle q_1^2 \rangle \simeq 2R_{max}^2 \sum_{j=\ell+1}^{K} \dot{\rho}_j^2 \tag{IV-25c}$$

Table (IV-1)

R^2	UPPER BOUND for $\langle y_j^2\rangle/\sigma_n^2$ for $R^2 \geq 1$
1	1.27411
4	1.44897
9	1.47941
16	1.48833
25	1.49248
36	1.49477
49	1.49609
64	1.49695
81	1.49753
100	1.49792

There is some computational evidence that $\langle q_1^2 \rangle \approx 60R_{max}^2$ and further investigation of the validity of this simple approximation will be pursued. Continuing with the evaluation of the elements of Φ leads to

$$\langle q_2^2 \rangle = \frac{1}{\sigma_n^4} \{ \sum_{j=1}^{\ell} p_j^2 P_j^2 (1 + R_j^2) + \sum_{j=\ell+1}^{K} p_j^2 \sigma_n^2 \} \tag{IV-26a}$$

$$\approx \frac{1}{\sigma_n^2} \{ \sum_{j=1}^{\ell} 2p_j^2 R_j^2 (1 + R_j^2) + \sum_{j=\ell+1}^{K} p_j^2 \} \tag{IV-26b}$$

$$\approx \frac{4}{\alpha^2} \sum_{j=1}^{\ell} R_j^4 (1 + R_j^2) + \frac{2}{\alpha^2} \sum_{j=\ell+1}^{K} R_j^2 \tag{IV-26c}$$

$$\approx \frac{2}{\alpha^2} \sum_{j=\ell+1}^{K} R_j^2 = \frac{1}{\sigma^2} \sum_{j=\ell+1}^{K} p_j^2 \tag{IV-26d}$$

and for the present study p_j and $\dot{p}_j$ will be assumed to have the required symmetry to have $\langle q_1 q_2 \rangle = 0$. None of the forms of equations (IV-25) or (IV-26) present significant computational difficulty and the sensitivity of system performance to the accuracy of approximating these terms will be investigated via simulations. Returning to the estimate $\hat{e}$ of (IV-1)

$$\hat{e} = \Phi^{-1} \Lambda_0 = \begin{pmatrix} \langle q_1^2 \rangle & 0 \\ 0 & \langle q_2^2 \rangle \end{pmatrix}^{-1} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \tag{IV-27a}$$

$$= \begin{pmatrix} \dfrac{1}{\langle q_1^2 \rangle} & q_1 \\ \dfrac{1}{\langle q_2^2 \rangle} & q_2 \end{pmatrix} \tag{IV-27b}$$

and the individual error estimates are

$$\hat{e}_\theta = \frac{1}{\langle q_1^2 \rangle} q_1 = \frac{1}{\langle q_1^2 \rangle} \cdot \frac{\hat{\alpha}}{\hat{\sigma}_n^2} \sum_{j=1}^{K} \hat{p}_j \{ V_j \frac{I_1(\ )}{I_0(\ )} - \hat{P}_j \} \tag{IV-28a}$$

$$\hat{e}_\alpha = \frac{1}{<q_2^2>} q_2 = \frac{1}{<q_2^2>} \cdot \frac{1}{\hat{\sigma}_n^2} \sum_{j=1}^{K} \hat{p}_j \{V_j \frac{I_1(\ )}{I_0(\ )} - \hat{P}_j\} \tag{IV-28b}$$

where the "^" indicates parameter evaluated at the estimate from the previous scan, e.g., $\hat{P}_j = \hat{\alpha}\hat{p}_j = \hat{\alpha}p[\theta_A(\tau_j) - \hat{\theta}(t_k)]$. The above error may be used to update the tracking receiver estimates of the angular position of the A/C, θ, and the peak amplitude α of the direct path signal.

Note that if the sums over samples for which $R_j^2 < 1$, i.e., $1 \le j \le \ell$, are neglected in computing $<y_j^2>$, the resulting equations for $\hat{e}_\theta$ and $\hat{e}_\alpha$ are

$$\hat{e}_\theta = \sum_{j=1}^{K} \hat{p}_j \{V_j \frac{I_1(\ )}{I_0(\ )} - \hat{P}_j\} \Big/ 2\hat{\alpha} \sum_{j=\ell+1}^{K} \hat{\dot{p}}_j^2 \tag{IV-29a}$$

$$\hat{e}_\alpha = \sum_{j=1}^{K} \hat{p}_j \{V_j \frac{I_1(\ )}{I_0(\ )} - \hat{P}_j\} \Big/ \sum_{j=\ell+1}^{K} \hat{p}_j^2 \tag{IV-29b}$$

where the denominator sums are nearly constant, i.e., ℓ constant, over a fairly wide range of R_{max}^2 and thus may be precomputed if on-line computational power is limited.

Preliminary results of simulations are presented in Figure IV-1. It should be noted that, at this point, no attempt has been made to assess the accuracy of the approximations in $<q_1^2>$ and $<q_2^2>$ or the effects of these inaccuracies on the performance of the estimation algorithms (sensitivity analysis). Again it should be pointed out that the error statistics (one σ values) given here are for the raw error which contains none of the smoothing anticipated when a tracking algorithm is in use e.g. those suggested in Section II of this report.

As was pointed out in Sections II and III, if $e = 0$ the residual mean square error associated with the estimate $\hat{e}$ is Φ_0^{-1}, i.e.,

$$<\hat{e}\hat{e}^T> \Big|_{e=0} = \Phi_0^{-1} = \begin{pmatrix} \frac{1}{<q_1^2>} & 0 \\ 0 & \frac{1}{<q_2^2>} \end{pmatrix} \tag{IV-30}$$

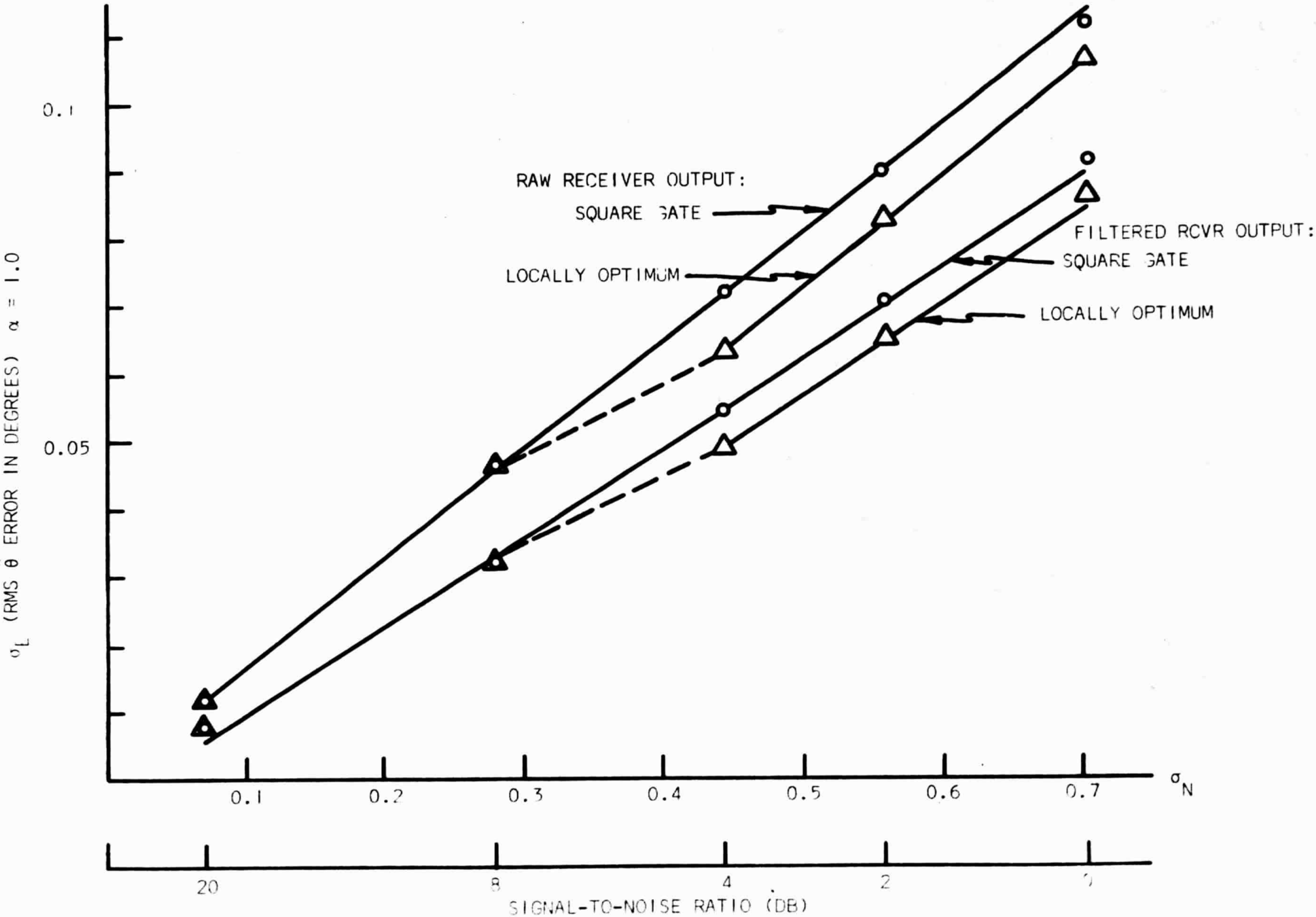


Figure IV-1 Error Performance of Simulated Receivers

and, therefore, $<\hat{e}^2_\theta> = \frac{1}{<q_I^2>}$. Using the simple approximation following (IV-25) yields

$$<\hat{e}^2_\theta> \simeq \frac{1}{60R^2_{max}} \qquad \text{(IV-31a)}$$

or

$$<\hat{e}^2_\theta>^{\frac{1}{2}} \simeq \frac{0.13}{R_{max}} \qquad \text{(IV-31b)}$$

for theoretical root-mean-square values of 0.013 and 0.051 for R^2_{max} of 100 (20db) and 6.3 (8db) respectively. These numbers are very close to those obtained in simulation (0.012 and 0.047, see Figure IV-1) and tend to show agreement between these theoretical developments and computer simulations. However, as the simulation results appear to produce smaller root-mean-square errors than the theory would indicate possible, there appears to be some need for further refinements of this analysis. Additionally, the simulations produce a significant bias in $\hat{e}_\theta$ which is, of course, counter to the theoretical development of an unbiased estimator, i.e., $<\hat{e}_\theta> = 0$. There is some evidence that this bias is due to a very remote characteristic of the specific computer program being used to generate the random noise in the simulation. This problem is now under study and alternate noise generating programs are being considered.

Comparison of I-F & Envelope Detectors

From Section III Equation (III-14), the mean square error of the i-f detector is given by

$$<\hat{e}^2_\theta>_{IF}\Big|_{e=0} = \frac{2N_0}{\hat{\alpha}^2\int_0^T \dot{\rho}^2[\theta_A(\tau)]d\tau} \qquad \text{(IV-32)}$$

which can be closely approximated by

$$<\hat{e}^2_\theta>_{IF} = \frac{2N_0}{\hat{\alpha}^2 \Delta\tau \sum_{j=1}^{K} \dot{p}_j^2} = \frac{2N_0 B}{\alpha^2} \cdot \frac{1}{\sum_{j=1}^{K} \dot{p}_j^2} \tag{IV-33a}$$

$$= \left(\frac{\hat{\alpha}^2}{\sigma_n^2} \sum_{j=1}^{K} \dot{p}_j^2\right)^{-1} = \left(2R^2_{max} \sum_{j=1}^{K} \dot{p}_j^2\right)^{-1} \tag{IV-33b}$$

where $\Delta\tau = B^{-1}$ is the reciprocal of the i-f bandwidth and $2N_0B = \sigma_n^2$. Comparing this with the results for the envelope processor, equation (IV-25c),

$$<\hat{e}^2_\theta>_E \Big|_{e=0} = <q_1^2>^{-1}$$

$$\simeq \left(2R^2_{max} \sum_{j=\ell+1}^{K} \dot{p}_j^2\right)^{-1} \tag{IV-34}$$

and the ratio for the two cases becomes

$$<\hat{e}^2_\theta>_{IF} / <\hat{e}^2_\theta>_E \simeq \sum_{j=\ell+1}^{K} \dot{p}_j^2 \Big/ \sum_{j=1}^{K} \dot{p}_j^2 \leq 1 \tag{IV-35}$$

For large signal-to-noise ratios, i.e., $R^2_{max} >> 1$, $\sum_{j=\ell+1}^{K} \dot{p}_j^2 \approx \sum_{j=1}^{K} \dot{p}_j^2$ and the mean-square performance of the two algorithms become nearly equal, e.g., for $R^2_{max} = 100$ (20db) the ratio is 0.94. Also the numerator of (IV-35) is a lower bound on the actual value, and yet it can be shown by actual computations that the value of the ratio of mean-square errors is bounded above by unity for R^2_{max} sufficiently large, e.g., $R^2_{max} \geq 6.3$ (8db). As the performance of the two processors is very nearly the same for reasonably high signal-to-noise ratios, all future efforts will be concentrated on the envelope processor which is much simpler to implement. One plausible explanation for this perhaps surprising result is that there is no information concerning the A/C angle in the phase of the i-f signal as modeled. A brief elementary study showed that, in the far field of the antenna, this signal model was appropriate.

Plans for Future Work

Work will continue in many of the areas discussed in this section with respect to the envelope detector processor, e.g. parametric sensitivity study via simulations, further analytic refinement of the bounds on the formulation of $\langle y_j^2 \rangle$, etc. New work will include analytic and simulation effort in the following areas:

1. Multipath suppression including use of both random process and unknown deterministic signal models of the multipath,
2. Revise signal model and resulting processor derivation to include logarithmic i-f as well as alternate types of detectors, e.g., linear/logarithmic squared amplitude envelope,
3. Investigate the feasibility of making the envelope processor adaptive to the antenna selectivity function; also look for robust designs which might be insensitive to this possibly variable signal feature without significant loss of error performance, e.g., the square gate receiver used in our simulation.

Most of this work is scheduled for completion prior to the start of programming the microprocessor, i.e., June 1976. All of this portion of the project should be completed by mid-July 1976.

V. CURRENT INVESTIGATIONS, SYSTEM CONSIDERATIONS AND RECOMMENDATIONS

All effort currently is related to optimal envelope processing. Theoretical investigations have been discussed and are summarized below:

1. Multipath-adaptive processing of the received signal envelope.
2. Comparative evaluation of finte-word length digital processors designed to accept linear envelope, log envelope and squared amplitude envelope signals.
3. Comparative evaluation of the layered tracking algorithm discussed with structures of the fully-recursive design.
4. Tentative consideration of algorithmic requirements for a beamwidth-adaptive feature.

Simulation work will follow the theoretical studies listed; in addition the receiver evaluation filters will be added to the simulation to facilitate evaluation and comparison of results with those of other studies.

System design of the prototype signal processor to be flight-tested has begun. Microcomputing equipment has been ordered, though previously unanticipated delivery delays has forced some change in the project schedule; a revised schedule is given in this report. A longer period of more extensive pre-flight exercising of the system than originally scheduled is desired, and it appears this may not be possible in the current funding period. This is discussed further below.

In general the software in the microcomputer must accomplish four distinct tasks:

1. Input conditioning and storage
2. Algorithmic calculations
3. Output updating and posting
4. Allocation of the machine resources to the above 3 tasks (i.e., the executive program).

The input conditioning and storage functions will be served by a direct-memory-access (DMA) controller, which will autonomously sample the input analog (envelope) signal over the proper intervals (determined by the prediction $\hat{\theta}(k|k-1)$ during the TO and FRO scans, perform analog-to-digital

conversion of these samples and then store the data in prescribed blocks of memory (RAM) in the machine. An interrupt from the DMA controller following storage of data for the TO and FRO scans each will enable processing of the new data (when otherwise timely).

The algorithmic calculations use only memory for source and sync of data and hence will run in the "background" under executive control (of initiation).

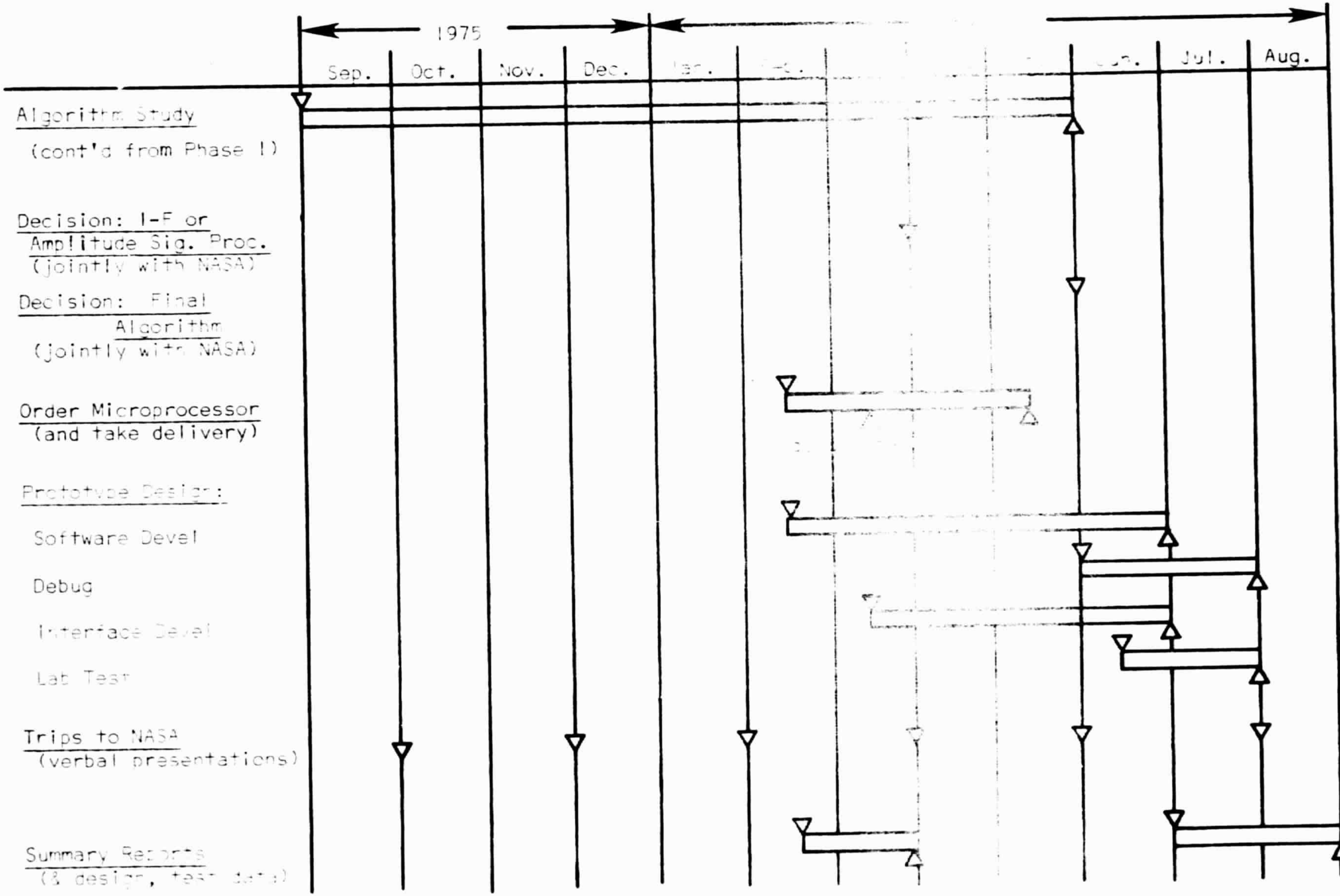
The output estimate is obtained by "marking" the extrapolating estimate at the occurrence of the Barker correlation peak (t_o pulse) that indicates the arrival of the next scan. A default trigger will provide this function (with suitable advisement) in the event of loss of sync signal. The angular coordinate estimate is then passed both to a serial output port for external distribution and to a parallel latched output port for local digital display.

The executive program takes all external timing and channel identification information and performs the scheduling function in the machine, including for example:

1. "Mark" the extrapolating estimate and terminate extrapolation when the new sync pulse arrives (if sync signal is lost, mark-by-default the extrapolating estimate but do not terminate extrapolation). Output the "marked" value of the angular coordinate estimate.
2. Adapt the algorithm to the elevation or azimuth channel, as appropriate to the upcoming scan;
3. Initiate processing of TO scan data after storage is complete.
4. Initiate processing of FRO scan data and subsequent estimate extrapolation after storage of FRO scan data is complete.
5. Resolve ambiguities in timing and syncing associated with turn-on, signal loss and recovery, etc.
6. "Idle" when all scheduled calculations are complete.

If the input data rate (multichannel) is too high, the executive program should also selectively edit the input data stream in an acceptable manner to not exceed the processor thru-put rate.

Above, the advisability of a somewhat fuller lab testing and pre-flight exercising of the prototype was mentioned (in connection with computer delivery delays and associated schedule impacts). The principal reason for this is that the prototype system, as it is evolving, will involve many different functions, only a few of which (the algorithmic calculations) will it have been feasible to test in the large FORTRAN simulation. It would seem prudent, if possible, to test <u>all</u> functions prior to interfacing the prototype with the Phase III Receiver. Relevant also is the available electrical and functional output of the Phase III Receiver, its full definition, and the potential necessity to install line drivers in the Phase III Receiver enclosure to send signals by coaxial cable to the prototype system. Our recommendations for future work are essentially that provisions be made for these tasks, particularly the system test. A real-time simulation of the Phase III Receiver can be economically developed in the microcomputer development system in the lab and used to exercise the completed prototype system in toto through its designed hardware interface. This would help to insure a smooth integration with the Phase III Receiver and to guarantee the expected performance at flight test time. A proposal for project continuation along these lines will be submitted.


1975
Sep.
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Algorithm Study
(cont'd from Phase I)
Decision: I-F or
Amplitude Sig. Proc.
(jointly with NASA)
Decision: Final
Algorithm
(jointly with NASA)
Order Microprocessor
(and take delivery)
Prototype Design:
Software Devel
Debug
Interface Devel
Lab Test
Trips to NASA
(verbal presentations)
Summary Reports

REFERENCES

1. G. A. McAlpine, J. H. Highfill, III, S. H. Irwin, Jr., J. E. [illegible]. "Optimization of MLS Receivers for Multipath Environments," Report No. EE-4033-101-75, Research Laboratories for the Engineering Sciences, School of Engineering and Applied Science, University of Virginia, Charlottesville, Virginia, December 1975.

2. James T. Murphy, Jr., "Locally Optimum Estimation with Applications to Communication Theory," Ph. D. dissertation, University of Florida, 1968.

3. G. J. Bierman, "Fixed Memory Least Squares Filtering," IEEE Transactions on Information Theory, IT-21:6, November 1975, pp. 690-692.

4. L. W. Nelson, E. Stear, "The Simultaneous On-Line Estimation of Parameters and States in Linear Systems," IEEE Transactions on Automatic Control, AC-21:1, February 1976, pp. 94-98.

5. B. T. Fang, "A Nonlinear Counterexample for Batch and Extended Sequential Estimation Algorithms," IEEE Transactions on Automatic Control, AC-21:1, February 1976, pp. 138-139.

6. W. B. Davenport, Jr. and W. L. Root, Introduction to Random Signals and Noise, New York: McGraw-Hill, 1958.

7. D. Middleton, An Introduction to Statistical Communication Theory, New York: McGraw-Hill, 1960.

8. H. L. Van Trees, Detection, Estimation and Modulation Theory, Part I, New York: Wiley & Sons, 1968.

9. J. Omura, T. Kailath, "Some Useful Probability Distributions," Report SU-SEL-65-079, Stanford Electronics Laboratories, Stanford University, Stanford, California, September 1965.